For Administration Use *(To be completed by the TA / invigilator)*

Session **A**: 15:00–17:00 ☐ Session **B**: 17:00–19:00 ☐

The University of Toronto Mississauga and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of a test and/or for revealing the test materials to other students or to any unauthorized institution.

Please note, once this test has begun, you ***CANNOT*** *re-write it.*

Session A: 15:00–17:00

Question A1 (5 points)

Let $\mathcal{F}$ be the set of all functions $\mathbb{R} \to \mathbb{R}$ and $\mathcal{D} \subseteq \mathcal{F}$ be the set of all *differentiable* functions. Define the relation $\sim$ on $\mathcal{D}$ by

$$f \sim g := \exists c \in \mathbb{R}. \forall x \in \mathbb{R}. f(x) = g(x) + c$$

Show that $\sim$ is an equivalence relation on $\mathcal{D}$. Prove that the map $d : \mathcal{D}/_\sim \to \mathcal{F}$ given by $d([f]) = f'$ is well-defined and injective.

Session B: 17:00–19:00

Question B1 (5 points)

Let $\mathcal{F}$ be the set of all functions $\mathbb{R} \to \mathbb{R}$ and $\mathcal{D} \subseteq \mathcal{F}$ be the set of all *differentiable* functions. Define the relation $\sim$ on $\mathcal{D}$ by

$$f \sim g := \exists c \in \mathbb{R}. \forall x \in \mathbb{R}. f(x) = g(x) + c$$

Show that $\sim$ is an equivalence relation on $\mathcal{D}$. Prove that the map $d : \mathcal{D}/_\sim \to \mathcal{F}$ given by $d([f]) = f'$ is well-defined and injective.

Session A: 15:00–17:00

Question A2 (5 points)
Compute

$$\int \frac{1}{x(1+\ln x)(1+\ln(1+\ln x))}\mathrm{d}x$$

Session B: 17:00–19:00

Question B2 (5 points)
Compute

$$\int \frac{e^{\sqrt{1+x^2}}}{\sqrt{1+x^2}}x\mathrm{d}x$$

Session A: 15:00–17:00

Question A3 (5 points)
Compute

$$\int \frac{1}{x\sqrt{1-(\ln x)^2}}\mathrm{d}x$$

Session B: 17:00–19:00

Question B3 (5 points)
Compute

$$\int \frac{1}{x(1+(\ln x)^2)}\mathrm{d}x$$