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*Please note, once this test has begun, you **CANNOT** re-write it.*

Question 1 (5 points)

Prove that

$$\sum_{n=1}^{\infty} \frac{n}{2^n} = 2$$

Hint: Differentiate $\sum_{n=0}^{\infty} x^n$.

Question 2 (5 points)

Prove that

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)(2n+2)} = \ln 2$$

Hint: Integrate $\sum_{n=0}^{\infty} (-x)^n$.

Question 3 (5 points)

Let K be a compact set. Let $f : K \rightarrow \mathbb{R}$ be a continuous function and $f_n : K \rightarrow \mathbb{R}$ be a sequence of continuous functions such that $\forall x \in K. f_n(x) \searrow f(x)$. Prove that $f_n \rightrightarrows f$.

Hint. Consider the sets $E_n = \{x \in K : f_n(x) < \varepsilon\}$.
