For Administration Use *(To be completed by the TA / invigilator)*

Session **A**: 09:00–11:00 ☐ Session **B**: 13:00–15:00 ☐

The University of Toronto Mississauga and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of a test and/or for revealing the test materials to other students or to any unauthorized institution.

Please note, once this test has begun, you ***CANNOT*** *re-write it.*

Writing Instruction You may find some of the following properties of the Riemann integral useful for some of the questions on the test. You may use any of them without proof.

Proposition 0.1 (Linearity). *For any $f, g \in \mathfrak{R}[a, b]$ and $c \in \mathbb{R}$, we have*

$$\int_a^b (f + cg) = \int_a^b f + c \int_a^b g$$

Proposition 0.2 (Subnormality). *For any $f \in \mathfrak{R}[a, b]$, we have*

$$\left| \int_a^b f \right| \leq \int_a^b |f|$$

Proposition 0.3 (Monotonicty). *If $f, g \in \mathfrak{R}[a, b]$ and $f \leq g$ (that is, $f(x) \leq g(x)$ for all $x \in [a, b]$), then*

$$\int_a^b f \leq \int_a^b g$$

Proposition 0.4 (Additivity). *If $f \in \mathfrak{R}[a, b]$ and $f \in \mathfrak{R}[b, c]$, then $f \in \mathfrak{R}[a, c]$ and*

$$\int_a^c f = \int_a^b f + \int_b^c f$$

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Question A1 (5 points)

Let $S = \bigcup_{k=1}^{n} I_k$ be a *disjoint* union of intervals $I_k = [a_k, b_k]$. Let $\chi_S(x) = \begin{cases} 1 & x \in S \\ 0 & x \notin S \end{cases}$ be the characteristic function of S. Show that for any $a, b \in \mathbb{R}$ such that $S \subseteq [a, b]$, we have

$$\int_a^b \chi_S = \sum_{k=1}^{n} (b_k - a_k)$$

Question B1 (5 points)

Same as **A1.**

Session B: 13:00–15:00

Session A: 09:00–11:00

Question A2 (5 points)

Let $f : [a, b] \to \mathbb{R}$ such that f is bounded and $f \in \mathfrak{R}[t, b]$ for all $t \in (a, b)$. Prove that $f \in \mathfrak{R}[a, b]$ and $\int_a^b f = \lim_{t\to a^+} \int_t^b f$.

Be careful! You need to show both the integral and limit exist.

Session B: 13:00–15:00

Question B2 (5 points)

Let $f : [a, b] \to \mathbb{R}$ such that f is bounded and $f \in \mathfrak{R}[a, t]$ for all $t \in (a, b)$. Prove that $f \in \mathfrak{R}[a, b]$ and $\int_a^b f = \lim_{t\to b^-} \int_a^t f$.

Be careful! You need to show both the integral and limit exist.

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(Use this page for Question 2 if needed)

Session B: 13:00–15:00

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Session A: 09:00–11:00

Question A3 (5 points)
Compute

$$\int t^3(3t^2-4)^{5/2}\,\mathrm{d}t$$

Session B: 13:00–15:00

Question B3 (5 points)
Compute

$$\int \frac{1}{\sqrt{9x^2-36x+37}}\,\mathrm{d}x$$

(*Hint*: for this question, you may need to compute $\int secx\ \mathrm{d}x$. To do so, multiply by $\frac{\sec x+\tan x}{\sec x+\tan x}$ and make a substitution)

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(Use this page for Question 3 if needed)

Session B: 13:00–15:00

(Use this page for Question 3 if needed)