The University of Toronto Mississauga and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of a test and/or for revealing the test materials to other students or to any unauthorized institution.

Please note, once this test has begun, you ***CANNOT*** *re-write it.*

Question 1 (5 points)

Let $f \in \mathfrak{R}[a, b]$ be non-negative (i.e, $f(x) \geq 0$ for all $x \in [a, b]$). Show that if $\int_a^b f = 0$, then $f(x) = 0$ almost everywhere. Is the converse true?

Question 2 (5 points)

Suppose $f, g : [a,b] \to \mathbb{R}$ are continuous with $f \geq g$. Show that $f = g \iff \int_a^b f = \int_a^b g$

Question 3 (5 points)

Let $f : [a, b] \to \mathbb{R}$ be bounded. Show that if f is continuous almost everywhere on $[a, b]$, then f is Riemann integrable on $[a, b]$.

Hint. You can't construct the value of the integral, so you must the Cauchy criterion. Cover the discontinuities of f by a small open cover U. Choose a δ_x-neighborhood for each $x \in [a, b] \setminus U$ by continuity of f. The set of all these δ_x-neighborhoods alongside U form an open cover of $[a, b]$; apply compactness to reduce to a finite subcover and use it to define your δ. For any two δ-fine partitions, construct a common refinement which includes all endpoints of every interval in the finite subcover. Each subinterval in the common refinement will either be contained in U or one of the δ_x-neighborhoods; use this to obtain a bound on the difference of Riemann sums.

Note. This question is meant for you to complete the proof of the equivalent criteria for Riemann integrability from class. Do not use tools that would make that proof circular (e.g. Darboux integrals). You do not need any theorems other than the Cauchy criterion.

Extra space for Question 3 if needed.