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*Please note, once this test has begun, you **CANNOT** re-write it.*

Question 1 (5 points)

Find all continuous functions $f : [-1, 1] \rightarrow \mathbb{R}$ satisfying

$$\int_a^{x^2} f(t) \, dt = \int_a^x f(t) \, dt$$

for all $x \in [-1, 1]$, where $a \in [-1, -1]$ is some fixed constant.

Question 2 (5 points)

Let $f : [a, b] \rightarrow \mathbb{R}$ be continuously differentiable with $f(a) = 0$. Show that

$$\left| \int_a^b f(x) \, dx \right| \leq \frac{(b-a)^2}{2} \sup_{x \in [a, b]} |f'(x)|$$

Question 3 (5 points)

Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous satisfying for all $x \in \mathbb{R}$ there exists $\delta > 0$ such that for all $0 < h < \delta$, we have

$$\int_x^{x+h} f(t) dt = 0$$

Show that f is identically zero.
