The University of Toronto Mississauga and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of a test and/or for revealing the test materials to other students or to any unauthorized institution.

Please note, once this test has begun, you ***CANNOT*** *re-write it.*

Question 1 (5 points)

Let $f : \mathbb{R} \to \mathbb{R}$ be T-periodic and integrable over any interval. Define $g : \mathbb{R} \to \mathbb{R}$ by $g(x) = \int_x^{x+T} f(s)\,\mathrm{d}s$. Find $g'(x)$ for all $x \in \mathbb{R}$.

Question 2 (5 points)

Let $f \in \mathfrak{R}[a, b]$ and F be a primitive for f. Show that $\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$.

Hint: Do this from scratch; the FTC does not apply.

Question 3 (5 points)

Let $f : [a, b] \to \mathbb{R}$. Say f has *bounded variation* if

$$\sup_{\substack{\Gamma \in \Omega[a,b] \\ \Gamma = (a = x_0 < \cdots < x_n = b)}} \sum_{i=0}^{n-1} |f(x_{i+1}) - f(x_i)| < \infty$$

Say f is *absolutely continuous* if for all $\varepsilon > 0$, there exists $\delta > 0$, such that for any finite sequence of non-empty, disjoint intervals $(a_1, b_1), \ldots, (a_k, b_k)$, if $\sum_{i=1}^{k} |b_i - a_i| < \delta$, then $\sum_{i=1}^{k} |f(b_i) - f(a_i)| < \varepsilon$.

Show that if f is absolutely continuous, then f has bounded variation.