The University of Toronto Mississauga and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of a test and/or for revealing the test materials to other students or to any unauthorized institution.

Please note, once this test has begun, you ***CANNOT*** *re-write it.*

Question 1 (5 points)

Find all values of $c > 0$ for which the series

$$\sum_{n=1}^{\infty} \frac{n!}{(1+c)(2+c)\cdots(n+c)}$$

converges.

Question 2 (5 points)

For a positive integer $n \geq 1$, say n is a *palindrome* if the digits of n read the same forwards and backwards. That is, if n is written $d_1 d_2 \cdots d_k$ then $d_1 d_2 \cdots d_k = d_k d_{k-1} \cdots d_1$. For example, 1331 is a palindrome but 159 is not.

You may take as granted that the number of k-digit palindromes is $9 \cdot 10^{\lceil k/2 \rceil - 1}$. Decide whether the series $\displaystyle\sum_{n \text{ palindrome}} \frac{1}{n}$ converges or diverges.

Question 3 (5 points)

Let $A = \sum_{n=0}^{\infty} a_n$ be absolutely convergent. Let $B = \sum_{n=0}^{\infty} b_n$ be convergent.

Define $c_n = \sum_{k=0}^{n} a_k b_{n-k}$. Show that $\sum_{n=0}^{\infty} c_n = AB$.

Hint: Let A_n, B_n, C_n denote the partial sums of $(a_k), (b_k), (c_k)$. The goal is to prove $C_n \to AB$.

To this end, show that for all n, $C_n - AB = (A_n - A)B + \sum_{i=0}^{n} a_{n-i}(B_i - B)$ (you may take this for granted at the cost of a 2pt deduction).

For scratch work; this page will not be graded unless you clearly indicate otherwise.

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